LAU
الجامعة اللبنانية الأميركية
Lebanese American University

LAU
School of Engineering

EXAMINATION BOOKLET

Date: / / (dd / mm / yyyy)

Name: MEE241 Spring 2016 | ID. No.:

Subject: Exam 1 Answer Key. | Section:

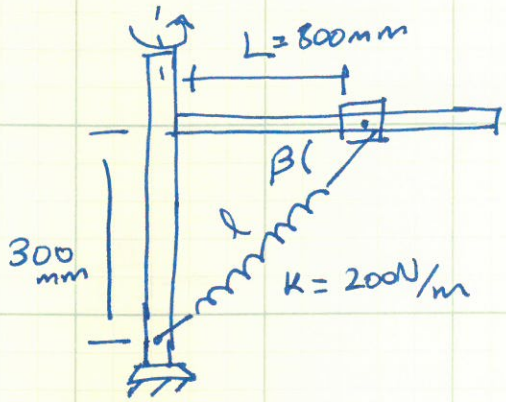
Instructor: | Box No.: | Email:

CLOSED-BOOK

Question:	1	2	3	4	5	6	7	8	9	10	Total:

Grade:

Problem 1 (30 pts)



spring force $= F_{sp} = K\Delta l$

$\Delta l = l - l_{unstretched}$

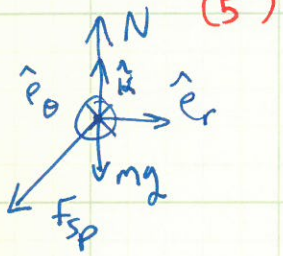
$= \sqrt{0.3^2 + 0.8^2} - 0.4$ (3)

$= 0.8544 - 0.4$

$= 0.4544$ m

(1) $\cos\beta = \frac{L}{l} = \frac{0.8}{0.8544} = 0.936$

F.B.D



(5)

(2) $\Sigma \vec{F} = m\vec{a}$

(3) $= m(a_r \hat{e}_r + a_n \hat{e}_n + a_z \hat{k})$

(2) $a_r = \ddot{r} - r\dot{\theta}^2$

(2) $a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

(1) $a_z = \ddot{z}$

we have $r = L$, $\dot{r} = \ddot{r} = 0$ (3)

$\dot{\theta} = \omega$, $\ddot{\theta} = 0$ (1)

$\ddot{z} = 0$ (no vertical motion) (1)

along $\hat{e}_r$: $-F_{sp}\cos\beta = m a_r$ (3)

$\Rightarrow -K\Delta l \cdot \cos\beta = m(-L\omega^2)$ (2)

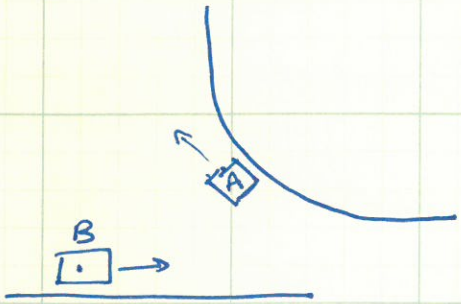
$\Rightarrow \omega^2 = 35.44 \Rightarrow \omega = 5.95$ rad/s (1)

Problem 2 (30 pts)

B moves on a straight line:

(2) $\vec{v}_B = 65\,\hat{i}$ (km/hr)

(2) $\vec{a}_B = 1200\,\hat{i}$ (km/hr²)



A moves on a circle:

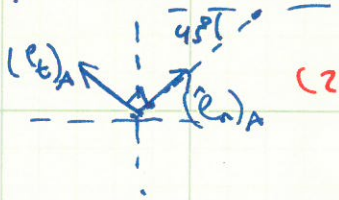
$\vec{v}_A = v_A(e_t)_A = 20(-\cos 45\,\hat{i} + \sin 45\,\hat{j})$ (km/hr) (2) (1)

choose  (2)

(4) $\vec{a}_A = a_t(e_t)_A + a_n(e_n)_A$

where $a_t = \frac{dv}{dt} = 0$ (2)

then:



(2)

(2) $a_n = \frac{v^2}{r} = \frac{(20)^2}{0.1} = 4000$ km/hr²

(1) then, $\vec{a}_A = 4000(\cos 45\,\hat{i} + \sin 45\,\hat{j})$ (km/hr²)

$(e_t)_A = -\cos 45\,\hat{i} + \sin 45\,\hat{j}$ (2)

$(e_n)_A = \cos 45\,\hat{i} + \sin 45\,\hat{j}$ (2)

we have:

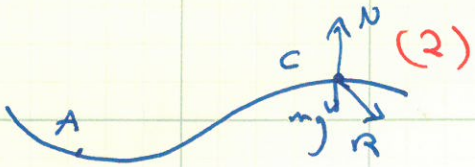
(2) $\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} \Rightarrow \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

(1) $= -79.14\,\hat{i} + 14.14\,\hat{j}$ (km/hr)

(2) $\vec{a}_A = \vec{a}_B + \vec{a}_{A/B} \Rightarrow \vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$

(1) $= 1628.4\,\hat{i} + 2828.4\,\hat{j}$ (km/hr²)

Problem 3: (20 pts)



(4) $\int_{S_A}^{S_C} a_t \, ds = \int_{V_A}^{V_C} v \, dv$

$a_t = cst$

(1) $\Rightarrow \quad a_t (S_C - S_A) = \frac{1}{2}(V_C^2 - V_A^2)$

(1) $\Rightarrow \quad a_t = 1.254 \ m/s^2$

$V_A = \frac{40}{3.6} = 11.11 \ m/s$

$V_C = \frac{90}{3.6} = 25 \ m/s$

(1) $\sum \vec{F} = m\vec{a}$

(3) $= m(a_t \hat{e}_t + a_n \hat{e}_n + a_z \hat{k})$

let the force by the road on the tires be $\vec{R} = R_t \hat{e}_t + R_n \hat{e}_n$

then, (2) $R_t = m a_t = 1504.8 \ N$

$R_n = m a_n = m \frac{V_C^2}{\rho_C} = 9375 \ N$
(2) (2)

the total force is $R = \sqrt{R_t^2 + R_N^2} = 9495 \ N$ (2)

Problem 4: (20 pts + 5 pts bonus)

F.B.D. (1)

↓ W

(1) $\Sigma \vec{F} = m\vec{a}$

along the vertical (1D motion):

(2) $-mg\frac{r_e^2}{y^2} = ma$

(1) $\Rightarrow$ $a = -\frac{gr_e^2}{y^2}$ here $a = \frac{d^2y}{dt^2} = \frac{dv}{dt}$

$v = \frac{dy}{dt}$

(4) $\int_{r_e}^{y} a\,dy = \int_{v_0}^{v} v\,dv$

(2) $-gr_e^2 \int_{r_e}^{y} \frac{1}{y^2}\,dy = \frac{1}{2}(v^2 - v_0^2)$ (1)

Note: we have: at $y = r_e$, $v = v_0$

(3) $gr_e^2\left(\frac{1}{y} - \frac{1}{r_e}\right) = \frac{1}{2}(v^2 - v_0^2)$

$\Rightarrow$ $v^2 = 2g\frac{r_e^2}{y} - 2gr_e + v_0^2$

(1) $\Rightarrow$ $v = \sqrt{2g\frac{r_e^2}{y} - 2gr_e + v_0^2}$

(b) maximum y $\Rightarrow$ v will switch sign (from +ve to -ve)

(2) $\Rightarrow$ $v = 0$

(1) $\Rightarrow$ $2g\frac{r_e^2}{y} - 2gr_e + v_0^2 = 0$

(1) $\Rightarrow$ $y_{max} = \frac{2gr_e^2}{2gr_e - v_0^2}$

bonus

(c) to escape: (3) $y_{max} \to \infty \Rightarrow v_0^2 \to 2gr_e$

$\Rightarrow v_{escape} = \sqrt{2gr_e}$ (2)